magnetic pressure must be taken into consideration in the computations.

The authors express gratitude to Yu. P. Raizer for interesting discussions of various aspects of the problems considered above.

LITERATURE CITED

1. A. S. Kompaneets, "Radio emission from an atomic explosion," Zh. Éksp. Teor. Fiz., 35, No. 6(12) 1538-1544 (1958).
2. V. Gilinsky, "Kompaneets model for radio emission from a nuclear explosion," Phys. Rev., 137, No. 1A, 50-55 (1965).
3. G. G. Vilenskaya, V. S. Imshennik, Yu. A. Medvedev, B. M. Stepanov, and L. P. Feoktistov, "Electromagnetic field excited in air by a nonstationary gamma-ray source located on an ideally conducting plane," Zh. Prikl. Mekh. Tekh. Fiz., No. 3, 18-26 (1975).
4. V. Gilinsky and G. Peebles, The development of a radio signal from a nuclear explosion in the atmosphere," J. Geophys. Res., 137, No. 1, 405-414 (1968).
5. J. R. Johler and J. C. Morgenstern, "Propagation of the ground wave electromagnetic signal with particular reference to a pulse of nuclear explosion," Proc. IEEE, 53, No. 12, 2048 (1965).
6. Yu. A. Medvedev, G. V. Fedorovich, and B. M. Stepanov, "Radio emission accompanying geomagnetic field perturbation by a gamma-ray source," Geomagn. Aéron., No. 2, 191-195 (1972).
7. H. L. Brode, "Gas dynamic motion with radiation," Astronaut. Acta, 14, 433 (1969).
8. W. J. Karzas and R. Latter, The electromagnetic signal due to the interaction of nuclear explosion with earth's magnetic field," J. Geophys. Res., 67, No. 12, 4635 (1962).
9. Yu. P. Raizer, "On deceleration and energy conversion of a plasma expanding into an empty space containing a magnetic field," Zh. Prikl. Mekh. Tekh. Fiz., No. 6, 19 (1963).
10. L. D. Landau and E. M. Lifshitz, Electrodynamics of Continuous Media, Addison-Wesley (1966).
11. G. V. Fedorovich, "Diamagnetism of conductors moving in a magnetic field," Zh. Prikl. Mekh. Tekh. Fiz., No. 2, 56 (1969).

ASYMPTOTIC ANALYSIS OF THE IGNITION OF A GAS BY A HEATED SURFACE

V. S. Berman and Yu. S. Ryazantsev

UDC 536.46

The problem of the ignition of a homogeneous hot mixture is a classical problem of combustion theory. Along with the practical significance, its analysis offers the possibility of working out approximate analytic and numerical methods of solution, using one of the simplest problems of nonsteady combustion as an example. The problem of the ignition of a condensed medium was first discussed in [1]. Gas ignition has been discussed numerically in a number of papers (for example, [2, 3] and the review [4]). Recently, efforts have been made to construct approximate analytic solutions of problems concerning ignition on the basis of the method of spliced asymptotic expansions. With the help of these methods an analysis has been carried out in [5, 6] of the ignition of a condensed phase by a luminous flux. The ignition of a condensed phase by a heated surface has been investigated in [7] by one of the authors.*

*V. S. Berman, "Some problems of the theory of the propagation of a zone with exothermic chemical reactions in gaseous and condensed media," Candidate's Dissertation, Institute of Applied Mechanics, Academy of Sciences of the USSR, Moscow (1974).

Moscow. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 1, pp. 68-73, January-February, 1977. Original article submitted January 26, 1976.

1. Statement of the Problem

The one-dimensional problem of the ignition of a gas by a flat heated surface kept at constant temperature can be described with a number of simplifying assumptions by the following system of equations and boundary and initial conditions:

$$\rho c \frac{\partial T}{\partial t'} = \frac{\partial}{\partial z}\left(\lambda \frac{\partial T}{\partial z}\right) - mc\frac{\partial T}{\partial z} + Qk\rho^n (1-y)^n e^{-E/RT}; \quad (1.1)$$

$$\rho \frac{\partial y}{\partial t'} = \frac{\partial}{\partial z}\left(D\rho \frac{\partial y}{\partial z}\right) - m\frac{\partial y}{\partial z} + k\rho^n (1-y)^n e^{-E/RT}; \quad (1.2)$$

$$\partial\rho/\partial t' + \partial m/\partial z = 0; \quad (1.3)$$

$$z = 0,\ T = T_w,\ \partial y/\partial z = 0 \quad (m(z=0,t')=0); \quad (1.4)$$

$$z = \infty,\ T = T_-,\ y = 0; \quad (1.5)$$

$$t' = 0,\ T = T_-,\ y = 0, \quad (1.6)$$

where T is the temperature, y is the concentration of the product of the reaction, ρ(T) is the density, λ is the thermal conductivity, c is the specific heat, z is the spatial coordinate, t' is the time, Q is the thermal effect of the reaction, k is the preexponential factor, E is the activation energy, n is the order of the reaction, R is the gas constant, D is the diffusion coefficient, m is the mass velocity of the gas motion, and T_w and T_- are the temperature of the wall and the initial temperature of the gas, respectively. It was assumed in the formulation of the problem that the pressure stays constant; this situation corresponds to numerical estimates [2]. Let us transform from the variables t', z and t', ψ by the formulas

$$m = -\partial\psi/\partial t',\ \rho = \partial\psi/\partial z,\ \psi(z=0,t') = 0.$$

Then we obtain from (1.1)-(1.3)

$$c\frac{\partial T}{\partial t'} = \frac{\partial}{\partial \psi}\left(\lambda\rho \frac{\partial T}{\partial \psi}\right) + Qk\rho^{n-1}(1-y)^n e^{-E/RT}; \quad (1.7)$$

$$\frac{\partial y}{\partial t'} = \frac{\partial}{\partial \psi}\left(D\rho^2 \frac{\partial y}{\partial \psi}\right) + k\rho^{n-1}(1-y)^n e^{-E/RT}. \quad (1.8)$$

For simplicity we assume that

$$\lambda\rho = \text{const},\ c = \text{const},\ D\rho^2 = \text{const}.$$

Let us transform to the dimensionless variables

$$X = \psi/\Delta x;\quad t = t'/\Delta t,\quad \Delta t = \frac{\beta\gamma}{\rho_w^{n-1} k e^{-\beta}},$$

$$(\Delta x)^2 = \lambda\Delta t/\rho_w c,\ \gamma \sim O(1),$$

$$L = \lambda/D\rho c,\ \gamma = c(T_w - T_-)/Q,\ \sigma = T_-/T_w - T_-,$$

$$\Theta = (T - T_-)/(T_w - T_-),\ \Gamma(\Theta) = (\rho/\rho_w)^{n-1},\ \beta = E/RT_w.$$

In place of (1.7), (1.8), and (1.4)-(1.6) it is possible to write

$$\frac{\partial\Theta}{\partial t} = \frac{\partial^2\Theta}{\partial X^2} + \beta(1-y)^n \Gamma(\Theta) e^{\frac{\beta(\Theta-1)}{\Theta+\sigma}}; \quad (1.9)$$

$$\frac{\partial y}{\partial t} = L^{-1}\frac{\partial^2 y}{\partial X^2} + \beta\gamma(1-y)^n \Gamma(\Theta) e^{\frac{\beta(\Theta-1)}{\Theta+\sigma}}; \quad (1.10)$$

$$X = 0,\ \Theta = 1,\ \partial y/\partial X = 0; \quad (1.11)$$

$$X = \infty,\ \Theta = 0,\ y = 0; \tag{1.12}$$

$$t = 0,\ \Theta = 0,\ y = 0. \tag{1.13}$$

We will find the asymptotic solution of the problem corresponding to the initial stage of the process, assuming that $\beta \gg 1$ and L, γ, n, σ, Γ, and $d\Gamma/d\Theta \sim 1$.

2. Solution of the Problem

Let us represent the solution $\Theta(X, t)$ in the form of a sum

$$\Theta(X, t) = \Theta_i(X, t) + u(X, t), \quad \Theta_i = \mathrm{erfc}\,(X/2\sqrt{t}),$$

where Θ_i is the solution of the problem

$$\partial\Theta_i/\partial t = \partial^2\Theta_i/\partial X^2,\ \Theta_i(0, t) = 1,\ \Theta_i(\infty, t) = \Theta_i(X, 0) = 0.$$

Then the problem (1.9)-(1.13) takes the form

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial X^2} + \beta(1-y)^n\,\Gamma(\Theta)\exp\left(\frac{\beta(\Theta_i + u - 1)}{(\Theta_i + u + \sigma)}\right); \tag{2.1}$$

$$\frac{\partial y}{\partial t} = L^{-1}\frac{\partial^2 y}{\partial X^2} + \beta(1-y)^n\,\Gamma(\Theta)\exp\left(\frac{\beta(\Theta_i + u - 1)}{\Theta_i + u + \sigma}\right); \tag{2.2}$$

$$u(0, t) = (\partial y/\partial X)(0, t) = y(\infty, t) = u(\infty, t) = 0,$$
$$u(X, 0) = y(X, 0) = 0. \tag{2.3}$$

Let us distinguish two spatial regions: the inner region adjacent to the point X = 0, where we introduce the variable $x = X\beta$, and the remaining part — the outer region. The term which describes the chemical reaction in the outer region is exponentially small.

Equations (2.1) and (2.2) take the form

$$\frac{1}{\beta^2}\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + \frac{1}{\beta}(1-y)^n\,\Gamma\exp\left(\frac{\beta(u - x/\beta\sqrt{\pi t})}{1+\sigma}\right) + o\left(\frac{1}{\beta}\right); \tag{2.4}$$

$$\frac{1}{\beta^2}\frac{\partial y}{\partial t} = L^{-1}\frac{\partial^2 y}{\partial x^2} + \frac{1}{\beta}(1-y)^n\,\Gamma\exp\left(\frac{\beta(u - x/\beta\sqrt{\pi t})}{1+\sigma}\right) + o\left(\frac{1}{\beta}\right) \tag{2.5}$$

in the inner region.

The solution of the problem is constructed in each of the regions in the form of asymptotic expansions which are consistent with the initial conditions and which satisfy the conditions at X = 0 in the inner region and at X = ∞ in the outer region.

The inner and outer expansions should satisfy the splicing conditions [8, 9]. Analysis of different forms of the expansions shows that the solutions in the inner and outer regions should be sought in the form

$$u = u_1(x, t)/\beta + u_2(x, t)/\beta^2 + \ldots; \tag{2.6}$$
$$y = y_0(x, t) + y_1(x, t)/\beta + \ldots;$$

$$u = U_1(X, t)/\beta + U_2(X, t)/\beta^2 + \ldots;$$
$$y = Y_0(X, t) + Y_1(X, t)/\beta + \ldots. \tag{2.7}$$

Substituting (2.6) into (2.4) and (2.5), with (2.3) taken into account, we have

$$\partial^2 u^1/\partial x^2 + (1 - y_0)^n \exp\left((u_1 - x/\sqrt{\pi t})/(1 + \sigma)\right) = 0, \tag{2.8}$$

$$u_1(0, t) = u_1(x, 0) = 0;$$
$$\partial^2 y_0/\partial x^2 = 0,\ (\partial y_0/\partial x)(0, t) = 0; \tag{2.9}$$

$$L^{-1}\partial^2 y_1/\partial x^2 + \gamma(1-y_0)^n \exp((u_1 - x/\sqrt{\pi t})/(1+\sigma)) = 0,$$
$$(\partial y_1/\partial x)(0, t) = 0. \tag{2.10}$$

From (2.9) it follows that $y_0 = y_0(t)$; then the general solution of Eq. (2.8) has the form

$$u_1(x, t) = x/\sqrt{\pi t} + (1+\sigma)[-n\ln(1-y_0) + \ln C_2 - 2\ln \operatorname{ch}(C_1 + x\sqrt{C_2/2(1+\sigma)})]. \tag{2.11}$$

Here $C_1 = C_1(t)$ and $C_2 = C_2(t)$ are some functions of t. From the boundary condition at x = 0 we find

$$C_2/(1-y_0)^n = \operatorname{ch}^2 C_1,$$
$$C_1 = \ln[\sqrt{C_2/(1-y_0)^n} \pm \sqrt{[C_2/(1-y_0)^n] - 1}]. \tag{2.12}$$

For the splicing of the main terms of (2.6) and (2.7) we obtain from (2.11) an asymptotic expression for u_1 as $x \to \infty$:

$$u_1(x \to \infty, t) = x(1/\sqrt{\pi t} - \sqrt{2(1+\sigma)C_2}) + (1+\sigma) \times$$
$$\times [-n\ln(1-y_0) + \ln C_2 + 2C_1 + \ln 4] + o(1) = X\beta(1/\sqrt{\pi t} -$$
$$- \sqrt{2(1+\sigma)C_2}) + (1+\sigma)[-n\ln(1-y_0) + \ln C_2 + 2C_1 + \ln 4] + o(1). \tag{2.13}$$

Comparing (2.13) with the corresponding expansions (2.7), we obtain

$$C_2 = \tau_0/t;\ \tau_0 = (1/2)\pi(1+\sigma),$$
$$U_1(0, t) = f(t) = (1/2\pi\tau_0)[-n\ln(1-y_0) + \ln C_2 - 2C_1 + \ln 4]. \tag{2.14}$$

Equations (2.14) determine the form of the function (2.11) and of the boundary condition for $U_1(X, t)$. In order to satisfy the initial condition (2.8), it is necessary to select the plus sign in (2.12). The solutions in the outer region should satisfy the equations and boundary conditions

$$\partial U_1/\partial t = \partial^2 U_1/\partial X^2,$$
$$U_1(X \to 0, t) = f(t),\quad U_1(\infty, t) = U_1(X, 0) = 0;$$
$$\partial Y_0/\partial t = (\partial^2 Y_0/\partial X^2)L^{-1},$$
$$Y_0(X \to 0, t) = y_0(t),\quad Y_0(\infty, t) = Y_0(X, 0) = 0,$$

whence

$$U_1(X, t) = \frac{X}{2\sqrt{\pi}} \int_0^t f(t') \frac{\exp[-X^2/4(t-t')]}{(t-t')^{3/2}} dt';$$
$$Y_0(X, t) = \frac{XL^{1/2}}{2\sqrt{\pi}} \int_0^t y_0(t') \frac{\exp[-X^2 L/4(t-t')]}{(t-t')^{3/2}} dt'. \tag{2.15}$$

As $X \to 0$, we have from (2.15)

$$Y_0(X \to 0, t) = y_0(t) - \frac{X\sqrt{L}}{\sqrt{\pi}} \int_0^t \frac{y_0'(t')}{\sqrt{t-t'}} dt' = y_0(t) - \frac{x}{\beta}\sqrt{\frac{L}{\pi}} \frac{d}{dt} \int_0^t \frac{y_0(t')}{\sqrt{t-t'}} dt'. \tag{2.16}$$

Equation (2.16) represents Y_0 in a form convenient for splicing with $y_1(x, t)$. Integrating (2.10), it is possible to obtain

$$\frac{\partial y_1}{\partial x} = L\gamma\left[\frac{\partial u_1}{\partial x} - \frac{1}{\sqrt{\pi t}} + \sqrt{\frac{C_2}{\pi\tau_0}} \operatorname{th} C_1\right]. \tag{2.17}$$

We obtain an integral equation for $y_0(t)$:

$$\frac{d}{dt}\int_0^t \frac{y_0(t')}{\sqrt{t-t'}}\,dt' = \gamma\sqrt{L}\left[\frac{1}{\sqrt{t}} - \frac{1}{\sqrt{\tau_0}}\sqrt{\frac{\tau_0}{t} - (1-y_0)^n}\right] \tag{2.18}$$

from (2.17) with $x \to \infty$ and from (2.16), having used the splicing condition.

Let us introduce the new variable $\tau = t/\tau_0$ and $y_0(t) = z(\tau)$; then it is possible to write

$$\frac{d}{d\tau}\int_0^\tau \frac{z(\tau')\,d\tau'}{\sqrt{\tau-\tau'}} = (\gamma\sqrt{L})\left[\frac{1}{\sqrt{\tau}} - \sqrt{\frac{1}{\tau} - (1-z)^n}\right] \tag{2.19}$$

instead of (2.18). Equation (2.18) is applicable in the time interval $0 \le t \le t^*$ $[y_0(t^*) = 1 - (\tau_0/t^*)^{1/n}]$.

Let us determine the time dependence of the heat flux into the heated wall

$$\partial\Theta/\partial X|_{X=0} = \beta\partial\Theta/\partial x = -\sqrt{C_2/\pi\tau_0}\cdot\mathrm{th}(C_1) + o(1). \tag{2.20}$$

Adopting the instant at which the heat flux into the wall vanishes as the instant of ignition, we conclude from (2.20) that this time is equal to t^*.

We note that the time of ignition t^*, which is a function of the parameters τ_0, γ, L, n, and β, can in this approximation be expressed in the form of a function of just two variables:

$$t^*/\tau_0 = F(\varepsilon, n); \quad \varepsilon = \gamma\sqrt{L}, \quad \tau_0 = 1/2\pi(1+\sigma).$$

Equation (2.19) is valid at large but finite values of β if $\varepsilon = o(1)$ and $t^*/\tau_0\beta \ll 1$. Equation (2.19), written in the form

$$\frac{d}{d\tau}\int_0^\tau \frac{z(x)\,dx}{\sqrt{\tau-x}} = \varepsilon\frac{F(\tau, z(\tau))}{\sqrt{\tau}}, \tag{2.21}$$

was solved numerically.

Expressing $z(\tau)$ from (2.21), we have

$$z(\tau) = \frac{\varepsilon}{\pi}\int_0^1 \frac{F(\tau x, z(\tau x))}{\sqrt{x(1-x)}}\,dx.$$

Introducing $\tau_k = kh$, $k = 0, 1, 2, \ldots$, we have

$$z(\tau_k) = \frac{\varepsilon}{\pi}\int_0^1 \frac{F(\tau_k x, z(\tau_k x))}{\sqrt{x(1-x)}}\,dx.$$

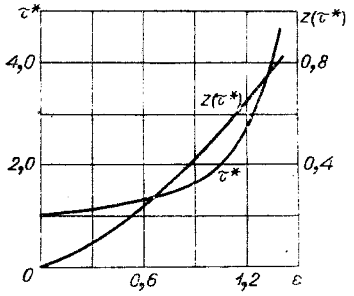

Fig. 1

Extrapolation between the nodes τ_{k-1}, and τ_k was carried out from the two preceding points τ_{k-1} and τ_{k-2}. The numerical solution of (2.19) for n = 1 is given in Fig. 1. As an example, let us discuss the case of the course of a zero-order reaction. The rate of the chemical reaction is equal to k exp (—E/RT) for $0 \leq y \leq 1$ and vanishes for $y > 1$. The solution of (2.19) is of the form

$$z(\tau) = \varepsilon[1 - (2/\pi)E(\sqrt{\tau})],$$

where E(x) is the complete elliptic integral. For $0 < \varepsilon \leq \pi/(\pi - 2) = 2.752$, the ignition time $\tau^* = 1$, and $z(\tau^*) = \varepsilon\,[(\pi - 2)/\pi]$. For $\varepsilon < 2.752$, the chemical reaction ceases at the instant $\tau = \tau^+$ $[z(\tau^+) = 1]$, where τ^+ is the root of the equation $1 = \varepsilon[1 - (2/\pi)E(\sqrt{\tau^+})]$.

LITERATURE CITED

1. Ya. B. Zel'dovich, "Contribution to ignition theory," Dokl. Akad. Nauk SSSR, 150, No. 2 (1963).
2. K. G. Shkadinskii and V. V. Barzykin, "Regularities of the ignition of gases by a heated surface with diffusion and hydrodynamics taken into account," Fiz. Goreniya Vzryva, No. 2 (1968).
3. A. G. Merzhanov, B. I. Khaikin, and K. G. Shkadinskii, "Establishment of steady-state propagation of a flame upon the ignition of a gas by a heated surface," Zh. Prikl. Mekh. Tekh. Fiz., No. 5 (1969).
4. A. G. Merzhanov and A. E. Averson, "Contemporary state of thermal ignition theory," Preprint Inst. Khim. Fiz. Akad. Nauk SSSR, Moscow (1970).
5. A. Linian and F. A. Williams, "Theory of ignition of a reaction solid by constant energy flux combustion," Sci. Technol., 3, 91 (1971).
6. A. Linian and F. A. Williams, "Radiant ignition of a reactive solid with in-depth absorption," Combust. Flame, 18, 85 (1972).
7. V. N. Vilyunov and R. S. Gol'dman, "Application of the method of spliced asymptotic expansions to one problem of ignition," in: Data of the Fourth Conference on Mathematics and Mechanics [in Russian], Izd. Tomsk. Univ., Tomsk (1974).
8. M. Van Dyke, Perturbation Methods in Fluid Mechanics, Academic Press (1964).
9. J. D. Cole, Perturbation Methods in Applied Mathematics, Blaisdell.

NUMERICAL STUDY OF THE EFFECT OF SURFACE THERMAL CONDITIONS ON FLOW IN THE BASE REGION OF A BODY OF FINITE DIMENSIONS

A. V. Babakov and L. I. Severinov

UDC 533.6.011

The results of the present work are obtained using the method of "fluxes" [1], which has properties of conservativity with respect to mass, momentum, and total energy. A characteristic feature of the method is the asymmetrical approximation of the convective terms.

It is assumed that the gas is Newtonian and perfect, has constant specific heat capacities, the coefficient of viscosity μ depends on the temperature in accordance with the law $\mu \sim T^{\omega}$ (ω = const), and the Prandtl number Pr is constant. Moreover, the Stokes hypothesis of the equality of the pressure and of the arithmetic mean of the three principal stresses with the opposite sign is satisfied. The calculations were carried out in the following coordinate system: the x axis is directed along the surface of the sphere, the y axis along the local normal to it, and the origin is placed at the leading critical point. We introduce the following notation: u and v are the velocity components along x and y; p, ρ, and T

Moscow. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 1, pp. 73–76, January–February, 1977. Original article submitted January 19, 1976.